MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT602

CourseName : Functional Analysis

Credits : 4 L - 3 T - 1 P - 0
Course Type : Core
Prerequisites : Real analysis, Linear algebra

Course Contents

Normed linear spaces and Banach spaces, Finite dimensional normed spaces, Riesz Lemma, compactness of closed unit disk, equivalence of norms, Schauder basis and separablilty. Bounded linear operators, continuous linear functionals, normed spaces of bounded linear operators, dual space, reflexive space and canonical mapping, weak and weak* topologies.

Hahn-Banach extension theorem, Uniform boundedness principle, open mapping theorem, closed graph theorem and their applications.

Inner product and Hilbert Spaces, orthogonal complements, Orthonormal sets, Total orthonormal sets, Bessel's Inequality, Parseval's relation, Riesz representation theorem. Adjoint operators in Hilbert Spaces, sesquilinear form, Compact Operators and Spectral Theorem, normal operator, unitary operators, Projections and orthogonal projections in Hilbert spaces, Best approximations in Hilbert Spaces.

Recommended Readings

Text books-

1. Nair M.T., *Functional Analysis: A first course*, PHI, 2002.
2. Kreyszig E., *Introductory functional analysis with applications*, Wiley publications, 1989.

Reference books-

1. Limaye, B.V., *Functional Analysis*, Wiley Publication, 1996.
2. Brown, A.L. and Page A., *Functional Analysis*, Van Nostrand Reinhold, 1970.
3. Kesavan S., *Functional Analysis*, Hindustan Book Agency, 2014.

Online resources-

1. NPTEL Course- https://nptel.ac.in/courses/111/105/111105037/
2. MITCourse- https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functional-analysis-spring-2009/

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

M. Sc. Electives II to IV Semester

S.No.	Course Code	Course Name	L	T	P	C
1.	21MAT801	Special Functions	3	0	-	3
2.	21MAT802	Applied Linear Algebra	3	0	-	3
3.	21MAT807	Commutative Algebra	3	0	-	3
4.	21MAT808	Field and Galois Theory	3	0	-	3
5.	21MAT809	Graphs and Matrices	3	0	-	3
6.	21MAT810	Advanced Functional Analysis	3	0	-	3
7.	21MAT811	Probability Theory	3	0	-	3
8.	21MAT812	Flow and Thermal Instability	3	0	-	3
9.	21MAT813	Fluid Dynamics	3	0	-	3
10.	21MAT814	Measure and Integration	3	0	-	3
11.	21MAT815	Advanced Analysis	3	0	-	3
12.	21MAT816	Introduction to Operator Theory	3	0	-	3
13.	21MAT817	Geometric Function Theory	3	0	-	3
14.	21MAT818	Fractional Calculus and its Application	3	0	-	3
15.	21MAT819	Calculus of variations and tensor analysis	3	0	-	3
16.	21MAT820	Fractional differential equations	3	0	-	3
17.	21MAT821	Dynamical Systems	3	0	-	3
18.	21MAT822	Numerical Linear Algebra	3	0	-	3
19.	21MAT823	Finite Element Method	3	0	-	3
20.	21MAT824	Integral Equations	3	0	-	3

Open Elective

S.No.	Course Code	Course Name	L	T	P	C
1.	21MAT803	Number theory	3	0	-	3
2.	21MAT804	Discrete Mathematics	3	0	-	3
3.	21MAT805	Classical Mechanics	3	0	-	3
4.	21MAT806	Introduction to Graph theory	3	0	-	3
5.	21MAT825	Integral Transforms	3	0	-	3

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT801

Course Name : Special Functions

Credits : 3 **L -** 3 **T -** 0 **P -** 0
Course Type : Elective
Prerequisites : Ordinary Differential equations, Differential and Integral calculus

Course Contents

The Gamma and Beta Function: Preliminaries, Euler's integral for Gamma (Z), Gamma and Beta functions and its elementary properties Factorial function, Gauss Multiplication formula, Legendre's duplication formula, Gauss multiplication theorem, Incomplete gamma function, Incomplete beta function, Mittag-leffler's function, Riemann Zeta Function.
The Hypergeometric Function : Definition,Integral representation of hypergeometric function, Transformations, Gauss's hypergeometric functions and its elementary properties, Gauss's hypergeometric differential equation and its solution, Gauss's hypergeometric function, Evaluation of hypergeometric function, relations of contiguity, theorem due to Kummer's, Generalized Hypergeometric series, the function of ${}_uF_v$, Bilateral hypergeometric series.
Kummer's function (The Confluent hypergeometric function): Definitions and some elementary results, Recurrence relations, The differential equation, Kummer's first and second formula, Addition and multiplication theorems, Integral representations, Basic properties of ${}_1F_1$, Special cases and its relation to other functions, Products of Kummer's functions.
Bessel functions: Bessel differential equation and its solution, Bessel's functions $J_u(x)$, recurrence relation, generating functions, integral representation, and orthogonality of Bessel functions, modified Bessel function and its properties**.**
Legendre functions : Legendre's differential equation and its solution, Relations between Legendre functions, The function P(x) and Q(x), Multiplications of two Legendre functions, Integral representations, Integrals involving Legendre functions, Associated Legendre functions.

Recommended Readings

Text book-

1. Rainville E.D., *Special Function*, The Macmillan Company, New York, 1960.
2. Sharma J.N. and Gupta R.K., *Special Functions*, Krishna's Educational Publishers..

Reference book-

1. Bell W.W., *Special Function for Scientists and Engineers*, D. Van Nostrand Company Ltd., 1968.
2. Andrews G.E., Askey R., Roy R., *Special Function*, Cambridge University Press, 1999.
3. Wang Z.X., Guo D.R., *Special Function*,World Scientific, 2010.
4. Prudnikov A.P., Brychkov Y.A., Marichev O.I., Gould G.G., *Integral and Series*, Gordon and Breach Science Publ., 1990.
5. Magnus W., Oberhettinger F., Soni R.P., *Formulas and Theorems for the Special Functions of Mathematical Physics*, 3rd Edition, Springer-Verlag Berlin Heidelberg GmbH, 1966.
6. Larry C.A, *Special Function of Mathematics for Engineers*, 2nd Edition, Oxford University Press, 1998.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT802

Course Name : Applied Linear Algebra

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Linear Algebra

Course Contents

Review on Inner Product Spaces, projection, orthogonal projection, best approximation, Finite Dimensional spectral theory for normal operators and self adjoint operators. Quadratic forms, Bilinear forms.

Orthogonal reduction, Discrete Fourier Transformation, Complementarity Subspaces, range-null space decomposition, orthogonal decomposition, Singular-value decomposition, Polar decomposition, Matrix Norms and location of eigenvalues, generalized inverse of a matrix.

PerronFrobenius Theory: Positive matrices, Nonnegative matrices, stochastic matrices and applications to Markov chains.

Recommended Readings

Text book-

1. Hoffman K., Kunze R., *Linear Algebra*, Prentice-Hall, Second Edition, 2008.
2. Meyer C.D., *Matrix Analysis and Applied Linear Algebra*, SIAM, 2001.
3. Horn R., Johnson C. R., *Matrix Analysis*, Cambridge University Press, New York, 1985.

Reference book-

1. Lay D.C., Lay S.R., McDonald J.J., *Linear Algebra and Its Applications*, Pearson, 2015.
2. Zhang F., *Matrix Theory-Basic Results and Techniques*, Second Edition, Springer, 2011.
3. Golub G.H., Van Loan C.F., *Matrix Computations*, Fourth Edition, Hindustan Book Agency, 2013.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT807

Course Name : Commutative Algebra

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Abstract Algebra

Course Contents

Revision of Rings and Ideals: Prime and maximal ideals. Chinese remainder theorem, Nilradical, Jacobson radical, operations on ideals, extension and contraction.

Basics of Modules: Module, submodule, quotient module, sums and products, Nakayama's lemma; Homomorphism, kernel, cokernel, direct sum, direct product, universal properties, free module, exact sequences, tensor product of modules and its exactness property, Localization of rings and modules.

Chain conditions: Noetherian rings, Hilbert basis theorem, Primary decomposition of ideals in Noetherian rings. Artinian rings and modules.

Integral Extensions: Integral dependence, going-up and going down theorems, Noether's normalization lemma, Hilbert Nullstellensatz, Krull's principal ideal theorem and dimension theorem.

Recommended Readings

Text book

1. Atiyah M.F. and MacDonald I. G., *Introduction to commutative algebra*, CRC Press, 2018.

Reference book

1. Matsumura H., *Commutative ring theory*, Cambridge Studies in Advanced Mathematics No. 8, Cambridge University Press, 1989.
2. Gopalakrishnan N.S., *Commutative Algebra*, Oxonian Press, 2015.

Online resources-

NPTEL Course: https://nptel.ac.in/courses/111/106/111106098/

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT808

Course Name : Field and Galois Theory

Credits : 3 **L -** 3 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Abstract Algebra

Course Contents

Field Theory: Review of basics of rings, integral domains and fields, Characteristic and prime subfields. Field extensions, Finite, algebraic, and finitely generated field extensions, Classical ruler and compass constructions, Splitting fields and normal extensions, algebraic closures. Finite fields. Quadratic and cyclotomic fields. Separable and inseparable extensions. Norm, trace and discriminant.

Galois Theory: Galois extensions, Galois groups, Fundamental Theorem of Galois Theory, Composite extensions, Examples (including cyclotomic extensions and extensions of finite fields). Solvability by radicals, Galois' Theorem on solvability.

Recommended Readings

Text book-

1. Artin M., *Algebra*, Prentice Hall of India, 1994.
2. Dummit D. S., Foote R. M., *Abstract Algebra*, 3rd Ed., John Wiley, 2004.

Reference book-

1. Gallian J.A., *Contemporary Abstract Algebra*, 9th Ed., Brooks/ Cole Cengage Learning, 2017.
2. Lang S., *Algebra*, 3rd Ed., Springer, New York, 2002.
3. Hungerford T.W., *Algebra*, Springer, 2003.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT809

Course Name : Graphs and Matrices

Credits : 3 L - 3 T - 0 P - 0

Course Type : Core / Elective

Prerequisites : Linear Algebra and Basic Graph Theory

Course Contents

Matrices, Eigenvalues of symmetric matrices, Graphs, Incidence matrix, Rank, Minors, Matchings in bipartite graphs. Adjacency Matrix, Eigenvalues of some graphs, Determinant, Bounds, Energy of a graph, Anti Adjacency matrix of a directed graph, Non-singular trees. Laplacian matrix, Basic properties, Computing Laplacian Eigenvalues, Matrix-tree theorem, Bounds of Laplacian spectral radius, Edge-Laplacian of a tree. Cycles and cuts, Fundamental cycles and Fundamental cuts, Fundamental matrices, Minors. Regular graphs, strongly regular graphs and friendship theorem, Graph with maximum Energy. Line Graph of tree, Block graphs, SignlessLaplacian matrix, Nullity of the line graph of a tree. Algebraic connectivity, classification of trees, Monotonicity properties of Fielder vector, Bounds of algebraic connectivity. Distance matrix of a graph, Distance and Laplacian matrix of a tree. Chordal graphs, positive definite completion.

Recommended Readings

Text book-

1. Bapat R., *Graphs and Matrices*, Text and Readings in Mathematics, Second Edition, Hindustan Book Agency, New Delhi, 2014.

Reference book

1. Cvetkovic D., Rowlinson.,Simic S., *An introduction to the theory of Graph Spectra,* First Edition, Cambridge University press, 2010.
2. Balakrishnan R., Ranganathan K., *A textbook of Graph Theory*, Second Edition, Springer, 2012.
3. Biggs N., *Algebraic Graph Theory*, Second Edition, Cambridge University press, 1993.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT810

Course Name : Advanced Functional Analysis

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Functional Analysis

Course Contents

Topological vector spaces (TVS): Definition of a TVS, Separation properties, balanced, convex and absorbing sets in TVS and their properties, special neighborhood bases, quotient and product spaces, finite dimensional TVS, linear maps.

Locally Convex Spaces (LCS): Definition of a locally convex space, special neighborhood bases, seminorms, Minkowski functional, relation between seminorms and locally convex spaces, metrizable LCS, bounded sets and their properties, normability of LCS, Hahn Banach theorem, Dual of a locally convex space, polars and their basic properties.

Topologies associated withpolars: weak and weak*-topologies, strong topology on X*, Banach-Alaoglu Theorem, Mackey topology and Mackey space, the topologies of bounded convergence and pointwise convergence.

Completeness in LCS: Barrelled and bornological spaces and their basic properties, equicontinuous family of linear maps between LCS,Banach–Steinhaustheorem, quasi-completeness, precompact sets, open mapping and closed graph theorems.

Recommended Readings

Text book-

1. Osborne M.S., *Locally Convex Spaces*, Springer, 2014.
2. Rudin W., *Functional Analysis*, McGraw Hill, Reprint 2018.

Reference book-

1. Narici L. and Beckenstein E., *Topological Vector Spaces*, 2nd edition, CRC Press, 2011.
2. Wilansky A., *Modern Methods in Topological Vector Spaces*, Dover Publication, 1984.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT811

Course Name : Probability Theory

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Measure and Integration

Course Contents

Algebra and σ-algebra, Borel σ-algebra, Lebesgue Measure, probability measure, basic laws of probability, Measurable functions, random variables, Monotone classes, π-systems, λ-systems. Monotone Class Theorem, Distribution function, inverse distribution function, the Fundamental Theorem of Probability, Lebesgue integration, Monotone convergence theorem, Fatou's Lemma, Dominating convergence theorem, Convergence "almost surely (a.s)", convergence in distribution, change of variable formula, expectation of a random variable, Product measure, Fubini's Theorem, independent random variables, Borel-Cantelli Lemmas, Tail σ-algebra, Kolmogorov's 0-1 Law. Various modes of convergence, law of large numbers, Characteristic functions, Moment expansion, characteristics function for Normal distribution. Inversion formula, density formula. Central Limit Theorems.

Recommended Readings

Text book-

1. Billingsley P., *Probability and Measure*, 3e, Wiley, 1995.
2. Athreya S. and Sunder V.S., *Measure and Probability*, Univ. Press, 2008.

Reference book-

1. Chung K. L., *A course in probability theory*, 3e, Academic Press, 2001.
2. Bhat B.R., *Modern Probability Theory*, 2nd edition, Wiley Eastern Ltd., 1985.
3. Parthasarathy K.R., *Introduction to Probability and Measure*, HBA, 2005.
4. Bartle R.G., *The elements of integration and Lebesgue Measure*, Wiley, 1966.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT812

Course Name : Flow and Thermal Instability

Credits : 3 **L -** 3 **T -** 0 **P -** 0
Course Type : Elective
Prerequisites : None

Course Contents

Basic concept of stability/instability - Static and dynamic instability - Temporal stability of inviscid incompressible flows - Squire transformation, Kelvin-Helmholtz stability - Temporal stability of viscous incompressible flows: Orr-Sommerfield problem, channel flow, plane Poiseuille flow, plane Couette flow, Blasius boundary layer flows, unbounded flows - Spatial stability of incompressible flows, incompressible inviscid flows - Centrifugal Instability - Thermal and inertial instabilities: Development of perturbations in space and time, equation of perturbation energy, the Boussinesq equations, free-free boundaries, rigid-rigid boundaries, free rigid boundaries - Convective instabilities: Benard problem, Couette-Benard flow, Rayleigh-Benard convection, Rayleigh-Taylor instability, Marangoni instability, Rayleigh-Benard-Marangoni instability, instabilities in stratified flows, introduction to chaos.

Recommended Readings

Text books-

1. CriminaleW.O., Jackson T.L., Joslin R.D., *Theory and Computations of Hydrodynamic Stability*, Cambridge University Press, 2018.
2. Chandrasekhar S., *Hydrodynamic and Hydromagnetic Stability*, Dover Publication.

Reference book-

1. Schmid P.J., Henningson D.S.,*Stability and Transition in Shear Flows*,Springer 2001

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT813

Course Name : Fluid Dynamics

Credits : 3 **L -** 3 **T -** 0 **P -** 0
Course Type : Elective
Prerequisites : None

Course Contents

Classification of fluids - Eulerian and Lagrangian approach - Irrotational flow - Equipotential surfaces - Mass flux density - Equation of continuity - Conservation of momentum - Integration of Euler's equation under different conditions - Bernoulli's equation - Kelvin's minimum energy and circulation theorems, Potential theorems - Complex potential, Sources, sinks, doublets and vortices, Milne-Thomson circle theorem - Blasius theorem - Three-dimensional flows - Motion of cylinders and spheres - Viscous flow, stress and strain analysis, Stokes hypothesis, Navier−Stokes equations of motion - Steady flow between parallel plates, Poiseuille flow, Steady flow between concentric rotating cylinders.

Recommended Readings

Text book-

1. Chorlton F., *Text Book of Fluid Dynamics*, CBS Publisher, 2005.

Reference books-

1. Fox R.W., Pritchard P. J. and McDonald A.T., *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.
2. Kundu P.K., Cohen I.M., Dowling D.R., *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT814

Course Name : Measure and Integration

Credits : 3 **L -** 3 **T -** 0 **P -** 0
Course Type : Elective
Prerequisites : Real Analysis

Course Contents

Review ofRiemann integrals, Insufficiency of Riemann integrals and the need of more general integrals, Measurable spaces and Measurable sets, Measurable functions and their properties, Measures, Lebesgue and Borel measures, Lebesgue Integration, Fatou's Lemma, Monotone Convergence Theorem, Dominate Convergence Theorem, Comparison of Lebesgue and Riemann Integrals, L_p Spaces, Various Modes of Convergences and their relations, Outer Measures and Generation of Measures, CarathéodaryExtenstion Theorem, Product Measures, Monotone class theorem, Tonelli's and Fubini's Theorem.

Recommended Readings

Text books-

1. Bartle R.G., *The Elements of Integration and Lebesgue Measures*, Wiley Publication, 1966.
2. Royden H.L., *Real Analysis*, 3rd Ed., Prentice Hall of India, 1988.

Reference books-

1. Halmos P.R., *Measure Theory*, Springer-Verlag, New York, 1974.
2. Kesavan S., *Measure and Integration*, TRIM, 2019.
3. Rudin W., *Real and Complex Analysis*, 3e, TMH, 1987.
4. Stein E.M. and ShakarchiR.,*Real Analysis- Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, 2005.

Online resources-

1. https://onlinecourses.nptel.ac.in/noc20_ma02/preview

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT815

Course Name : Advanced Analysis

Credits : 3 **L -** 3 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Functional Analysis, Measure and Integration

Course Contents

Review of general positive measure and integrals, Signed Measure, Hahn and Jordan Decomposition Theorems, The Radon-Nikodym Theorem, Complex measures and Radon-Nykodym theorem for complex measures, Total variation norm. L_p spaces, Vitali's convergence theorem, Dual of L_p spaces for $1 \leq p < \infty$, linear functionals on L_p-spaces, Chebychev's inequality. Regular Borel measure, Positive linear functionals on $C_c(X)$ for a locally compact Hausdorff space X, Riesz representation theorems, VitaliCarathe'odory theorem, Denseness of $C_c(X)$ in $L_p(X)$ for $1 \leq p < \infty$; Riesz-Thorin interpolation theorem.

Recommended Readings

Text books-

1. Rudin W., *Real and Complex Analysis*, 3rdedition, TMH, 1987.
2. Aliprantis C.D. and Burkinshaw O., *Principles of Real Analysis*, 3rd edition, Academic Press, 1998.
3. Lieb E.H. and Loss M., *Analysis*, 2nd edition, AMS, 2001.

Reference books-

1. Kesavan S., *Measure and Integration*, TRIM, Hindustan Book Agency, 2019.
2. Halmos P.R., *Measure Theory*, Springer-Verlag, New York, 1974.
3. Royden H.L., *Real Analysis*, 3rd Ed., Prentice Hall of India, 1988.
4. Stein E.M. and Shakarchi R., *Real Analysis- Measure Theory, Integration, and Hilbert Spaces*, Princeton University Press, 2005.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Mathematics

Course Code : 21MAT816

Course Name : Introduction to Operator Theory

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Functional Analysis

Course Contents

Compact Operators and its Properties, Spectral Results for Banach Space operators: Eigen spectrum and spectrum; spectral radius formula; spectral mapping theorem, Riesz-Schauder theory for compact operators. Operators on Hilbert spaces: self-adjoint, normal, unitary operators, Hilbert Schmidt operators. spectrum of various operators on Hilbert spaces, spectral theorem for compact self-adjoint operators; singular value decomposition of compact operators, Spectral theorem for self-adjoint operators, Solution of operator equations.

Recommended Readings

Text books-

1. Nair M.T., *FunctionalAnalysis: A First Course*, PHI-Learning, New Delhi, 2002 (Fourth Print: 2014).
2. LimayeB.V., *Functional Analysis*, 2nd Edition, New Age International, 1996.

Reference books-

1. Kreyszig E., *Introduction to Functional Analysis with Applications*, John Wiley & Sons, 1978.
2. Nair M.T., *Linear Operator Equations: Approximation and Regularization*, World Scientific, 2009.
3. Riesz F. and Nagy B.S., *Functional Analysis*, Dover Publications, 1990.
4. Conway J.B., *A Course in Functional Analysis*, 2nd Edition, Springer, 1990.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT817

CourseName : Geometric Function Theory

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Complex analysis

Course Contents

Analytic functions, Entire and meromorphic functions, Harmonic functions, Univalent functions.

Maximum modulus theorem, Schwarz lemma, Schwarz function, Weierestrass factorization theorem, Mittag-Leffler theorem, Picards theorems, subordination.

Families of analytic functions: Convex, Starlike, Spirallike, Geometric properties of functions, Conformal mapping on simply connected domains, Mapping properties of special functions, Riemann mapping theorem, Schwarz-Christoffel transformations, Potential function, Laplace equation and solution.

Recommended Readings

Text book-

1. Graham I. and Kohr G., *Geometric Function Theory in One and Higher Dimensions*, Marcel Dekker Inc., New York, 2003.
2. Krantz S.G., *Complex variables*, Chapman & Hall/CRC, Indian Reprint 2012.

Reference book-

1. Conway J.B., *Functions of One Complex Variable*, Springer Verlag, NewYork, 2nd Ed., 1978.
2. Ahlfors L.V., *Complex Analysis*, McGraw-Hill, 1966.
3. Rudin W., *Principles of Mathematical Analysis*, McGraw-Hill, 1976.
4. Nehari Z., *Conformal Mapping*, Dover publications, NewYork, 1952.
5. Lang S., *Complex Analysis*, Springer Verlag, NewYork, 4th ed. 1999.
6. Duren, P.L., Univalent Functions, Springer Verlag, NewYork, 1935.
7. Hille, E., Analytic Function Theory (Vol. II), 2nd Ed., Chelsea Publications, 1987.

Online resources-

1. Youtube lectures by Bonfert Taylor

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT818

Course Name : Fractional Calculus & its Application

Credits : 3 **L -** 3 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Ordinary and partial differential equations, Differential and Integral calculus

Course Contents

Brief review of Special Functions of the Fractional Calculus**,** Definition of Mittag-Leffler Functions of one and two parameters, Relations of Mittag-Leffler Function to some other functions, The Laplace transform of Mittag-Leffler Function in two parameters. Wright Function, Definition of Wright function, Integral relation and relation to other functions, Miller Ross function.

The Riemann Liouville Fractional Integral, Fractional Integrals of some functions namely binomial function, exponential, the hyperbolic and trigonometric functions, Bessel's functions, Hyper-geometric function, Dirichlet's Formula, Laplace Transform of the Fractional integral, Leibnitz's Formula for Fractional Integrals. Derivatives of the Fractional Integral and the Fractional Integral of Derivatives.

Derivatives, Properties of Fractional derivatives, Riemann-Liouville fractional derivatives, Riemann-Liouville left-sided derivative, Riemann-Liouville right-sided derivative, Leibnitz's Formula of Fractional Derivatives. Laplace transform of fractional derivatives, Fourier transform of fractional derivatives and Mellin transform of fractional derivatives. Fractional derivatives of standard functions Left and right fractional derivatives.

Definition of Weyl Fractional Integral, Weyl Fractional Derivatives, A Leibniz Formula for Weyl Fractional Integral and simple applications.

Definition Caputo Fractional Derivative, Leibnitz's formula for Caputo fractional derivative, Laplace transform of Caputo fractional derivative, Difference between Caputo fractional derivative and R-L fractional derivative, Caputo left-sided derivative, Caputo right-sided derivative, Caputo-Fabrizio fractional derivative.

(No. of lectures-35)

Recommended Readings

Text Books-

1. Miller K.S. and Ross B., *An Introduction to the Fractional Differential Equations,* John Wiley and Sons, 1993**.**
2. Samko S.G., Kilbas A.A., Marichev O.I., *Fractional Integrals and Derivatives*, Gordon and Breach Science Publishers, 1987.
3. Kilbas A.A., Srivastava H.M., Trujillo J.J., *Theory and Applications of Fractional Differential Equations*, Elsevier, 2006.

Reference books:-

1. Oldham K.B. and Spanier J., *The Fractional Calculus*, Academic Press Inc., 1974.
2. Ricardo A., Dina T., Delfim F.M. Torres, *The Variable-Order Fractional Calculus of Variations*, Springer, 2019.
3. Podlubny I., *Fractional Differential Equation*, Academic Press Inc., 1999.
4. Das S., *Functional Fractional Calculus*, Springer 2011.

Online/E resources:-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : **Department of Mathematics**

Course Code : **21MAT819**

Course Name : **Calculus of Variations and Tensor Analysis**

Credits : 3 **L -** 0 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Calculus

Course Contents

Calculus of variations: Basic concepts of calculus of variations, Variation and its properties, Euler's equation, Fundamental lemma of calculus of variation, Functionals dependent on higher order derivatives and on several independent variables, Variational problem in parametric form, applications Variational problem with fixed boundaries, Variational problem with moving boundaries, Sufficient condition for an extremum. Isoparametric problem.

Tensor Analysis: Notation and Definitions of Contravariant and covariant tensors of first and second order. Invariants. Contravariant, covariant and mixed tensors. The Kronecker delta. Algebra of tensors Symmetric and skew-symmetric tensors. Addition and scalar multiplication. Contraction and Quotient law for tensors. Reciprocal Tensor. Definition and properties of first and second kind of Christoffel's symbols, covariant differentiation of tensor, Ricci's theorem, Riemann- Christoffel 's tensor and its properties, Covariant curvature tensor .

Recommended Readings

Text Books:-

1. Nayak P.K.*,Textbook of Tensor Calculus and Differential Geometry,* PHI Learning, 2012.
2. Gupta A.S.*,Calculus of Variations with applications,*PHI Learning, 2005.

Reference Books:-

1. Gelfand I.M. and Fomin S.V., *Calculus of Variations,*Dover Publications,2000.
2. Irgens F.*, Tensor Analysis,* Springer,2019.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT820

CourseName : Fractional Differential Equations

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Basic calculus, numerical methods

Course Contents

Linear Fractional Differential Equations: Fractional Differential Equation of a General Form. Existence and Uniqueness Theorem as a Method of Solution. Dependence of a Solution on Initial Conditions.

The Laplace Transform Method. Standard Fractional Differential Equations. Sequential Fractional Differential Equations.

Fractional Green's Function: Definition and Some Properties. One-Term Equation. Two-Term Equation. Three-Term Equation. Four-Term Equation. General Case: n-term Equation.

Other Methods for the Solution of Fractional-order Equations: Power Series Method. Babenko's Symbolic Calculus Method. Method of Orthogonal Polynomials. The Mellin Transform Method.

Numerical Evaluation of Fractional Derivatives: Approximation of Fractional Derivatives. The "Short-Memory" Principle. Calculation of Heat Load Intensity Change in Blast Furnace Walls. Order of Approximation. Computation of Coefficients. Higher-order Approximations.

Numerical Solution of Fractional Differential Equations: Initial Conditions: Which Problem to Solve? Numerical Solution. Examples of Numerical Solutions. The "Short-Memory" Principle in Initial Value Problems for Fractional Differential Equations. Matrix approach to discrete fractional calculus. Numerical solution of nonlinear problems.

Applications. Fractional order systems and controllers.

Recommended Readings

Text book-

1. Podlubny. I., *Fractional differential equations*, Academic Press, 1999.
2. Kilbas. A.A.,Srivastava, H.M., Trujillo, J.J., *Theory and applications of fractional differential equations*, Elsevier, 2006.

Reference book-

1. Miller K.S. and Ross B., *An introduction to the fractional calculus and fractional differential equations*, John Wiley and SOns, 1993.
2. Diethelm K., *The analysis of fractional differential equations*, Springer, 2004.
3. Zhou, Y., *Basic theory of fractional differential equations*, World Scientific, 2014.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT821

Course Name : Dynamical Systems

Credits : 3 **L -** 3 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Ordinary Differential Equations.

Course Contents

One-dimensional flows: fixed points and stability, linear stability analysis, Saddle- node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Flows on the circle. **Two-dimensional flows:** Linear systems, nonlinear autonomous systems, phase portraits, Fixed points and linearization, conservative systems, index theory, limit cycles, Poincare Bendixson theorem, Lienard systems, Saddle-node, Transcritical and Pitchfork bifurcations, Hopf bifurcation. Chaos.Discrete dynamical systems

Recommended Readings

Text book-

1. Strogatz S.H., *Nonlinear dynamics and Chaos,* Perseus books publishing, 1994.
2. Perko L., *Differential Equations and* Dynamical Systems*,* 3rd Edition, Springer (India), 2004.

Reference book-

1. Sandefur J.T., *Discrete dynamical systems Theory and applications,* Clarendon press, 1990.
2. Scheinerman E.R.*, Invitation to Dynamical Systems.* Dover Publications Inc., 2012.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT822

Course Name : Numerical Linear Algebra

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Linear Algebra and Numerical Analysis.

Course Contents

Review of basic concepts: Floating-point arithmetic, matrices, operation counts. Matrix multiplication, block matrices. Matrix norms. Linear system sensitivity, conditioning.

Matrix factorizations: Cholesky factorization. QR factorization. LU factorization and Gaussian elimination; partial pivoting. Complete pivoting, rook pivoting. Error analysis. Numerical examples.

Linear least squares problem: Basic theory using singular value decomposition (SVD) and pseudoinverse. Perturbation theory. Numerical solution: normal equations. SVD and rank deficiency.

Eigenvalue problem: Basic theory, including perturbation results. Power method, inverse iteration. QR algorithm. Arnoldi iteration, Lancoz method.

Iterative methods for linear systems: Review of Jacobi and Gauss-Seidel iterations. Residual corrector method. SOR iterations. Kronecker product. Krylov subspace methods, conjugate gradient method. GMRES. Preconditioning.

Recommended Readings

Text books:

1. Demmel J.W., *AppliedNumerical Linear Algebra*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 1997.
2. Trefethen L.N. and BauD.,*Numerical Linear Algebra*, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA.

Reference books:

1. Kincaid D. and Cheney W.*, Numerical Analysis: Mathematics of Scientific Computing,* 3rd Ed, Brooks/Cole (2002).
2. Datta B.N., *Numerical Linear Algebra and applications*, 2nd Edition, Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2010.

Online resources:

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : **Department of Mathematics**

Course Code : **21MAT823**

Course Name : **Finite Element Methods**

Credits : 3 L - 3 T - 0 P - 0

Course Type : Elective

Prerequisites : Basics of numerical methods

Course Contents

Overview of finite element method, comparison with finite difference methods, Weighted residual methods: Galerkin's, Least squares and Collocation methods. Shape functions, Normalised coordinates.

Variational methods: Functional and its variation, Rayleigh-Ritz method, Equivalence of Rayleigh-Ritz and Galerkin methods in one and two dimensions.

Finite element method: Applications to ordinary differential equations, Elliptic equation, Node-wise assembly, Higher order elements, Element of rectangular shape, Parabolic Equations in one and two dimensions, and Hyperbolic Equations.

Text Books:

1. Fish J. and Belytschko T., *A First Course in Finite Elements*, Wiley 2007.
2. Hughes T. J. R., *The Finite Element Method*, Prentice-Hall, 1986.

Reference Books:

1. Cook R. D., Malkus D. S., Plesha M. E. and Witt R. J., *Concepts and Applications of Finite Element Analysis*, 4th Edition, John Wiley, 2002.
2. Bathe K. J., *Finite Element Procedures*, PHI 2006.
3. Seshu P., *Textbook of Finite Element Analysis*, PHI Learning 2018.
4. Reddy J. N., *An Introduction to the Finite Element Method*, Third Edition, Tata McGraw-Hill 2014.
5. Shrikhande M., *Finite Element Method and Computational Structural Dynamics*, PHI Learning Pvt. Ltd., New Delhi, 2014.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT824

Course Name : Integral Equations

Credits : 3 **L -** 3 **T -** 0 **P -** 0

Course Type : Elective

Prerequisites : Basic theory of differential equations, Calculus of several variables.

Course Contents

Definition and classification, conversion of initial and boundary value problems to an integral equation, Eigen-Values and Eigen functions. Solutions of homogeneous and general Fredholm integral equations of second kind with separable kernels.

Solution of Fredholm and Volterra integral equations of second kind by methods of successive substitutions and successive approximations, Resolvent kernel and its results.

Integral equations with symmetric kernels: Complex Hilbert space, Orthogonal system of functions, fundamental properties of eigen values and eigen functions for symmetric kernels, expansion in eigen-functions and bilinear forms, Hilbert-Schmidt theorem.

Solution of Fredholm integral equations of second kind by using Hilbert-Schmidt theorem. Fredholm theorems. Solution of Volterra integral equations with convolution type kernels by Laplace transform.

Text book:

1. Wazwaz A. M., *Linear and Nonlinear Integral Equations Methods and Applications*, Springer-Verlag Berlin Heidelberg, 2011.
2. Kanwal R. P., *Linear Integral Equations*, Birkhäuser Basel, 2nd edition, 1997.

Reference Books:

1. Mikhlin, S. G., *Linear Integral Equations*, Routledge, 1961.
2. Masujima M., *Applied Mathematical Methods of Theoretical Physics - Integral Equations and Calculus of Variations*, Weinheim, Germany: Wiley-VCH, 2005.
3. Lovitte W. V., *Linear Integral Equations*, Dover Publications; Reissue edition, 2005.

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT803

Course Name : Number Theory

Credits : 3 L - 3 T - 0 P - 0

Course Type : Open Elective

Prerequisites : Knowledge of number system

Course Contents

Congruences: Basic definitions and properties, complete and reduced residue systems, theorems of Fermat, Euler & Wilson, application to RSA cryptosystem. Linear congruences and Chinese remainder theorem, quadratic congruences, and Quadratic Reciprocity law. **Arithmetical functions**: Euler's Function, τ and σ functions with some properties and their rate of growth, Möbius-Inversion formula.
Diophantine Approximation: Continued fractions and their connection with Diophantine approximations, applications to Pell's equations.
Diophantine Equations: Linear equations, Binary quadratic forms, Solutions of some quadratic and higher degree diophantine equations.
Partitions: Partitions of a number, Some basic properties and results.

Recommended Readings

Text books-

1. Burton D., *Elementary Number Theory*, McGraw Hill, 2006.
2. Jones G.A. and Jones J.M., *Elementary Number Theory*, Springer UTM, 2007.

Reference books-

1. Baker A., *A concise introduction to the theory of numbers*, Cambridge Univ. Press, 1984.
2. Hardy G.H. and Wright E.M., *An introduction to the theory of numbers*, 4th Editions, Oxford, Univ. Press, 1960.
3. Ireland K.F. and Rosen M.I., *A Classical Introduction to Modern Number Theory*, Springer, 1990.
4. Niven I., Zuckerman H.S. and Montgomery H.L., *An introduction to the theory of numbers*, 5e, Wiley, 1991.

Online resources-

1. https://nptel.ac.in/courses/111/103/111103020/

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT804

Course Name : Discrete Mathematics

Credits : 3 L - 3 T - 0 P - 0

Course Type : Open Elective

Prerequisites : none; [preferred – understanding of basic mathematics]

Course Contents

Logic: Propositional Logic, language of propositional logic, truth table, natural deduction, predicate logic: language of predicate logic, Logical inference with Quantifiers. Proof techniques: Introduction to different standard proof techniques.

Set Theory: Review of basic set operations, cardinality of a set. Countable and uncountable sets. Relations, Types of relations, operations of relations and applications, Poset, Congruence arithmetic. Partially ordered sets and Lattices, Hasse Diagrams, lattices as algebraic systems sub-lattices, direct product and Homomorphisms, Complete lattices.

Combinatorics: Counting techniques: Pigeon Hole principle, inclusion exclusion principle, recurrence relation and generating function.

Graphs: Complete graphs, regular graphs, bipartite graphs, Vertex degree, subgraphs, paths and cycles, the matrix representation of graphs, fusion, trees and connectivity, bridges, spanning trees, chromatic number, connector problems, shortest path problems, cut vertices and connectivity.

Recommended Readings

Text books:

1. Rosen K.H., *Discrete Mathematics and its Applications with Combinatorics and Graph Theory*, 7th Edition, Tata McGraw-Hill Edu. 2012.
2. Liu C.L. and Mohapatra D., *Elements of Discrete Mathematical*, 4th Ed., Tata McGraw-Hill, 2012.

Reference books:

1. West D. B., *Introduction to Graph Theory,* Second Edition, Pearson, 2001.
2. Kolman B., Busby R. and Ross S.C., *Discrete mathematical structures*, 4th edition. Prentice Hall of India, 2002.
3. Wilson R.J., *Introduction to Graph Theory*, Fourth Edition, Prentice Hall of India, 1996.
4. Bondy J.A. and Murty U.S.R., *Graph Theory*, Springer, 2008.
5. Sane S.S., *Combinatorial Techniques*, Hindustan Book Agency, 2013.

Online resource-

1. https://nptel.ac.in/courses/106/108/106108227/

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT805

Course Name : Classical Mechanics

Credits : 3 L - 3 T - 0 P - 0
Course Type : Open Elective
Prerequisites : None

Course Contents

D'Alembert's principle, General equations of motion, Motion of the centre of inertia (Motion of translation), Motion relative to the centre of inertia (Motion of rotation)
Motion about a fixed axis: Moment of effective forces about a fixed axis of rotation, Moment of momentum, Kinetic energy of a rotating body about a fixed line, Principle of angular momentum, Principle of energy and work.
Lagrange's equations of motion: Degrees of freedom and generalized coordinates, Lagrange's equations of motion under finite forces, Lagrangian function, Deduction of principle of conservation of energy from the Lagrange's equations.
Motion in three dimensions: Euler's theorem for motion in three dimensions, system of moving axes, Euler's dynamical equations of motion under finite forces, Kinetic energy of body rotating about a fixed point, Eulerian angles and Euler's geometrical equations, Deduction of Euler's dynamical equations from Lagrange's equations of motion.

Recommended Readings

Text books-

1. Rana N.C. and Joag P.S., *Classical Mechanics,* Tata McGraw-Hill, 2015.
2. Goldstein H., Poole C.P. and Safko J., *Classical Mechanics,* Pearson Education, 2011.

Reference book-

1. Hand L.N. and Finch J.D., *Analytical Mechanics,* Cambridge University Press, 2008.

Online resources-

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT806

Course Name : Introduction to Graph Theory

Credits : 3 L - 3 T - 0 P - 0

Course Type : Open Elective

Prerequisites : UG Mathematics

Course Contents

Definition and basic concepts, Trees, characterizations, counting of minimum spanning tree, graph and matrices, Paths and distance in graphs, center and median of a graph, activity digraph and critical path, Eulerian graphs, Definition and characterization, Hamiltonian graphs, Necessary and sufficient conditions, Planar Graphs: properties, dual, genus of a graph. Peterson graph.

Graph coloring, vertex coloring, chromatic polynomials, edge coloring, planar graph coloring, Matching and factorizations, maximum matching in bipartite graphs, maximum matching in general graphs, Hall's marriage theorem, factorization, Networks, The Max-flow min-cut theorem, connectivity and edge connectivity, Menger's theorem.

Recommended Readings

Text book-

1. West D. B., *Introduction to Graph Theory*, Second Edition, Pearson, 2001.

Reference book-

1. Bondy J. A. and Murty U. S. R., *Graph Theory*, Springer, 2008.
2. Wilson R. J., *Introduction to Graph Theory,* Fourth Edition, Prentice Hall, 1996.

Online resources-

1. Graph theory: NPTEL (https://nptel.ac.in/courses/111/106/111106102/)

MALAVIYA NATIONAL INSTITUTE OF TECHNOLOGY JAIPUR

Department/Centre : Department of Mathematics

Course Code : 21MAT825

Course Name : Integral Transforms

Credits : 3 L - 3 T - 0 P - 0

Course Type : Open Elective

Prerequisites : Ordinary Differential equations, Differential and Integral calculus

Course Contents

Laplace Transforms : Definition of Laplace Transform and examples, Sufficient conditions for the existence of Laplace Transform, Laplace Transform of some elementary, Properties of Laplace Transform, shifting and scaling properties, Differentiation and Integration properties of Laplace Transform, The Error function, Transform of Bessel function, Periodic functions, Initial & Final Value Theorem, Inverse Laplace Transforms and its examples, Convolution theorem and related examples.

Bilateral Laplace Transform, Relation between Bilateral Laplace transform and Laplace Transform, Some Properties.

Applications of Laplace Transform: Solution of ODE – Initial value problems for linear equations with constant coefficients, Solution of Simultaneous ODE with constant coefficient.

Fourier Transform: Fourier sine and cosine transformation and its examples, Properties of Fourier sine and cosine transform, Transform of Derivatives, Parseval's Theorem, Inversion theorem, Convolution theorem, Relation between Bilateral Laplace transform and Fourier Transform. Applications of Fourier Transform.

Hankel Transform : Elementary properties, Inversion theorem, transform of the derivative of functions, transform of elementary functions, Parseval's theorem relation between Laplace transform and Hankel transform, Basic operational property of Hankel transform

Mellin Transform: Definition and properties of Mellin transform, shifting and scaling properties, Mellin transform of derivative and integral.

Hartley Transform: Definition of Hartley Transform and examples, Inversion theorem, Relation between Stieltjes Transform and Fourier Transform.

Recommended Readings

Text Books:-

1. Debnath L., Bhatta D., *Integral transforms and their Applications*, CRC Press/Chapman and Hall, 3rd Edition, 2015.
2. Patra B., *An Introduction of Integral Transform*, Taylor & Francis/CRC, 2018
3. Drof R.C., ;*Transform and Applications*, Taylor & Francis/CRC, 3rd Edition, 2010

Reference books:-

1. Andrews L.C., Shivamoggi B.K., *Integral Transform for Engineers*, SPIE Optical Engineering Press, 1999.
2. Sneddon I.N., *The use of Integral Transform*, McGraw Hill, 2nd Edition, 1972.